

Chaotic Spiral Galaxies

G. Contopoulos · M. Harsoula

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Abstract We study the role of asymptotic curves in supporting the spiral structure of a N-body model simulating a barred spiral galaxy. Chaotic orbits with initial conditions on the unstable asymptotic curves of the main unstable periodic orbits follow the shape of the periodic orbits for an initial interval of time and then they are diffused outwards supporting the spiral structure of the galaxy. Chaotic orbits having small deviations from the unstable periodic orbits, stay close and along the corresponding unstable asymptotic manifolds, supporting the spiral structure for more than 10 rotations of the bar. Chaotic orbits of different Jacobi constants support different parts of the spiral structure. We also study the diffusion rate of chaotic orbits outwards and find that chaotic orbits that support the outer parts of the galaxy are diffused outwards more slowly than the orbits supporting the inner parts of the spiral structure.

Keywords galaxies: structure, kinematics and dynamics, spiral

1 Introduction

It is well known that the spiral arms of galaxies are density waves. This means that the spiral arms are not always composed of the same matter but they only represent the maxima of the density along every circle around the center. The stars passing through the spiral arms stay longer close to them, thus producing an increase of the local density.

The linear theory of spiral density waves assumes that the potential V , the density ρ (or the surface density σ) and the distribution function f (phase space density) have small deviations from their axisymmetric values (V_0, ρ_0, f_0) .

G. Contopoulos
Research Center for Astronomy, Academy of Athens, Soranou Efessiou 4, GR-115 27 Athens,
Greece
E-mail: gcontop@academyofathens.gr

M. Harsoula
Research Center for Astronomy, Academy of Athens, Soranou Efessiou 4, GR-115 27 Athens,
Greece
E-mail: mharsoul@academyofathens.gr

This linear theory was initiated by Lindblad (1940, 1942) but it was developed in its modern form by Lin and Shu (1964, 1966), Kalnajs (1971), Lynden-Bell and Kalnajs (1972) Toomre (1977), and others.

However the deviations near the main resonances of a galaxy (inner and outer Lindblad resonances and corotation) are large, thus near these resonances a nonlinear theory is necessary (Contopoulos 1970, 1973, 1975).

In both the linear and the nonlinear theory, whenever the perturbation is relatively small, chaos is unimportant. Although some chaotic orbits appear near all the unstable periodic orbits, their proportion and their effects are small. This is the case of most normal spirals, where the density perturbations are of order 2–10% of the axisymmetric background.

On the other hand in barred galaxies the perturbations are large, of the order of 50 – 100%. In such cases the chaotic orbits play an important role in the dynamics of the galaxies. The main chaotic effects in a galaxy appear near corotation.

It is well known now that chaos is generated by the overlapping of resonances (Contopoulos 1966, Rosenbluth et al. 1966, Chirikov 1979). In the region near corotation there are many resonances between the angular velocities of the stars ($\Omega - \Omega_s$) (in the frame rotating with the angular velocity of the pattern Ω_s) and the epicyclic frequency κ .

The ratio of these frequencies is

$$q = \frac{\Omega - \Omega_s}{\kappa} \quad (1)$$

These resonances are congested in a relatively small interval of Jacobi constants E_j around the Jacobi constant E_{j0} at corotation. Some important resonances in an extended region around corotation are $1/q = 4/1, 3/1, 2/1$ (inside corotation), $q = 0$ (at corotation) and $-2/1, -1/1$ (outside corotation). Chaos produced by the interactions of resonances, extends around the envelope of the bar and along the spiral arms beyond the end of the bar. The first example of a chaotic orbit that fills an envelope of the bar and the inner parts of the spiral structure was provided by Kaufmann and Contopoulos (1996) (Fig.1).

However the most surprising result came from N-body simulations, which indicated that most orbits along the spiral arms beyond the ends of the bar are chaotic (Voglis et al. 2006a).

Thus started a systematic study of the chaotic spiral arms outside corotation in strong barred galaxies (Voglis and Stavropoulos 2005, Voglis et al. 2006a, b, Romero-Gomez et al. 2006, 2007, Tsoutsis et al. 2008, Athanassoula et al. 2009a, b, **2010**, Har-soula et al. 2009, 2011). In the present paper we present the most recent results of this study.

2 Chaotic Density Waves

Although the spiral arms beyond the ends of the bars are composed of chaotic orbits, nevertheless these spiral arms are density waves, i.e. the density maxima are populated by different stars at every time.

The density of the stars is maximum in areas where their velocities are minimum. This happens mainly near the apocentres and the pericentres of the orbits. And these

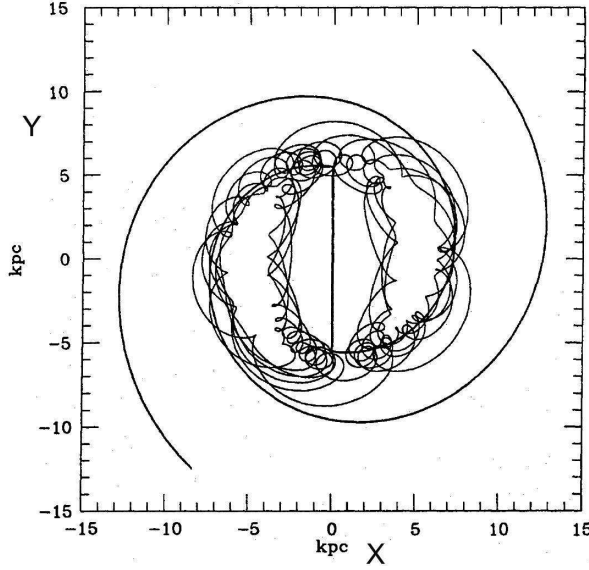


Fig. 1 First example of a chaotic orbit in a galactic model of NGC 3992 in [10].

apocentres and pericentres appear close to the asymptotic manifolds of the main unstable periodic orbits around corotation.

The most important families of unstable periodic orbits in the corotation region are the families PL_1, PL_2 around the unstable Lagrangian points L_1 and L_2 .

In a simple model of a barred galaxy the Hamiltonian near corotation (r_*) (Contopoulos 1978) is given by the relation

$$H = h_* + \kappa_* I_1 + a_* I_1^2 + 2b_* I_1 I_2 + c_* I_2^2 + A_* \cos 2\theta_2 = h \quad (2)$$

where h is the Jacobi constant, I_1 the epicyclic action, $I_2 = J - J_*$ is the azimuthal action, (where J, J_* are the angular momenta of the star and of corotation) θ_2 is the azimuth of the star and $h_*, \kappa_*, a_*, b_* < 0, c_* < 0, A_*$ are constants. The action I_1 is also constant, because the conjugate angle θ_1 , does not appear in the Hamiltonian. The orbits PL_1, PL_2 are represented by the unstable equilibrium points of the system, namely when

$$\partial H / \partial I_2 = \partial H / \partial \theta_2 = 0 \quad (3)$$

Therefore

$$\sin 2\theta_2 = 0 \quad (4)$$

and

$$b_* I_1 + c_* I_2 = 0 \quad (5)$$

The orbits close to the Lagrangian points L_1, L_2 have $\theta_2 = \pi/2$ and $\theta_2 = 3\pi/2$, (while the orbits close to the Lagrangian points L_4, L_5 have $\theta_2 = 0$, and $\theta_2 = \pi$),

In the lowest approximation the orbits PL_1, PL_2 start with $\theta_2 = \pi/2$, or $\theta_2 = 3\pi/2$, and

$$h_* + \kappa_* I_1 - A_* = h \quad (6)$$

hence

$$I_1 = \frac{h - h_* + A_*}{\kappa_*} > 0 \quad (7)$$

Therefore these orbits exist whenever $h > h_* - A_*$.

After finding I_1 we can find I_2 from Eq.(5).

We have (Contopoulos 1975)

$$b_* = \frac{\Omega_* \kappa_*'}{r_* \kappa_*^2}, c_* = \frac{\Omega_* \Omega_*'}{r_* \kappa_*^2} \quad (8)$$

where Ω is the angular velocity, and κ is the epicyclic frequency. The subscript $*$ denotes the values at corotation and the accents mean derivatives with respect to the radius r . Thus Eq.(5) can be written

$$\kappa_*' I_1 + \Omega_*' I_2 = 0 \quad (9)$$

We consider now a simple model of the form

$$V_o' = \frac{c^2}{r^\rho} \quad (10)$$

where V_o is the potential and V_o' is the force as a function of r . Then we have

$$\Omega = \sqrt{\frac{V_o'}{r}} = \frac{c}{\sqrt{r^{\rho+1}}} \quad (11)$$

and

$$\kappa = \sqrt{V_o'' + \frac{3V_o'}{r}} = \frac{c}{\sqrt{r^{\rho+1}}} (3 - \rho) \quad (12)$$

Thus Eq.(9) gives

$$(3 - \rho) I_1 + I_2 = 0 \quad (13)$$

and if $\rho < 3$ we find that I_2 is negative. In particular in a Keplerian model $\rho = 2$ and the relation (13) becomes

$$I_1 + I_2 = 0 \quad (14)$$

Therefore $I_2 < 0$. The initial point of the orbit PL_1 (with $\theta = \pi/2$ and increasing θ) is $P\bar{L}_1$ inside the corotation distance with

$$\Delta r_o = \frac{2\Omega_*}{r_* \kappa_*^2} I_2 < 0 \quad (15)$$

The orbit PL_1 is described counterclockwise (while the galaxy is rotating clockwise.) Thus the orbit PL_1 intersects the $\theta = \pi/2$ axis once more with $\Delta r_o > 0$ and decreasing θ (Fig. 2). The asymptotic manifolds from the orbits PL_1 , PL_2 on the configuration plane are shown in Fig.2. There are two unstable manifolds from PL_1 namely U along a trailing spiral and UU along the leading edge of the bar, and two stable manifolds, SS (along a leading direction) and S (along the trailing edge of the bar). These manifolds contain successive apocentres ($\dot{r}=0$) of the asymptotic orbits i.e. orbits starting on this manifold. Similar manifolds emanate from $P\bar{L}_1$, $P\bar{L}_2$, corresponding to the pericentres of the asymptotic orbits.

The orbits starting close to PL_1 but not exactly on the asymptotic manifold approach this point along the stable directions S and SS and then deviate along the

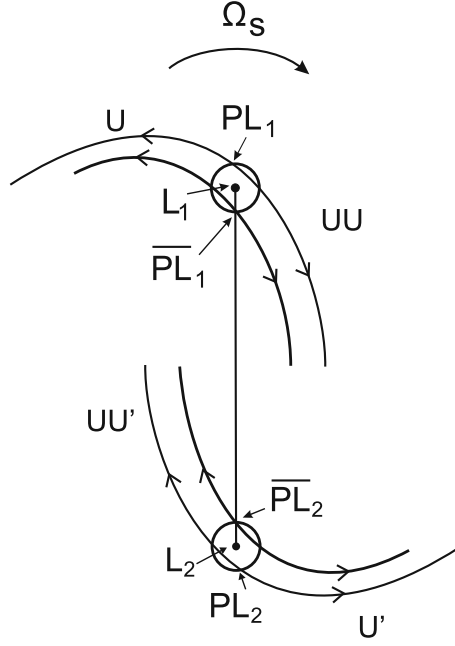


Fig. 2 The asymptotic manifolds from the orbits PL_1 , PL_2 on the configuration plane.

unstable directions U and UU . After a longer time these orbits form trailing spiral arms and the envelope of the bar. The forms of the asymptotic curves emanating from PL_1 and PL_2 are shown in Fig. 3. We see that the curve U from PL_1 reaches the neighborhood of PL_2 , making oscillations around the asymptotic curve SS' of larger and larger amplitude as it approaches PL_2 , coming closer and closer to the asymptotic curves U' and UU' from PL_2 , which are symmetric to U and UU with respect to the center of the galaxy. Thus the matter that starts close to PL_1 and moves along the asymptotic curve U , after reaching the neighborhood of PL_2 moves very close to the asymptotic curves U' and UU' and approaches again the neighborhood of PL_1 . In a similar way matter moves along the asymptotic curves UU , U' and UU' . Thus we have circulation of the material along the spiral arms that lasts for a substantial fraction of the Hubble time, until the spiral arms fade away. The individual orbits along the asymptotic curve U or close to it, are of the form of Fig. 4. The orbits start by making some rotations close to the periodic orbit PL_1 and they deviate away from PL_1 , reaching the neighborhood of PL_2 . Then they proceed either close to the spiral U' (Fig. 4a), or close to the envelope of the bar UU' (Fig. 4b).

The unstable asymptotic curves of the various unstable periodic orbits for the same Jacobi constant cannot cross themselves or each other. Thus they are obliged to follow nearly parallel paths. The main unstable orbits inside corotation are the families 4/1, 3/1 and 2/1 (inner Lindblad), while close to corotation the most important families are PL_1 and PL_2 whose asymptotic manifolds are shown in Fig. 5a. On the other hand, all the manifolds of the unstable periodic orbits corresponding to the same Jacobi constant, are approximately parallel and contribute to the formation of spiral arms outside corotation (Fig. 5b). This is the phenomenon of coalescence that was described by Tsoutsis et al.(2008). If we consider also similar figures for other values of the

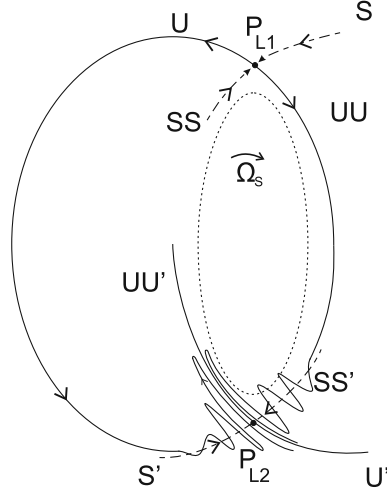


Fig. 3 The forms of the asymptotic curves emanating from PL_1 and PL_2 .

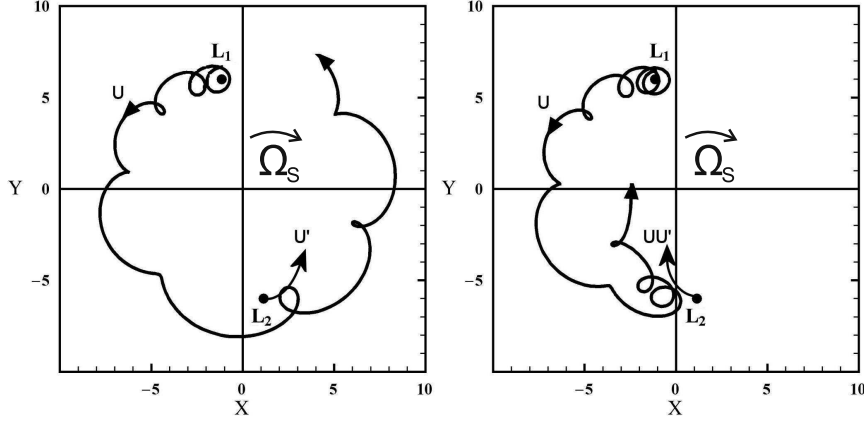


Fig. 4 Orbits starting close to PL_1 (around the Lagrangian point L_1) approach the Lagrangian point L_2 and then deviate (a) along U' , or (b) along UU' .

Jacobi constant (in which case overlapping of various curves is permitted) we see the formation of thick spiral arms. In fact the observed spiral arms in N-body simulations of barred galaxies are very close to the general form of the spiral arms produced by the superposition of the various asymptotic curves.

However orbits close to the particular unstable periodic orbits support particular parts of the spiral arms. Thus it is of interest to study the orbits close to every resonance.

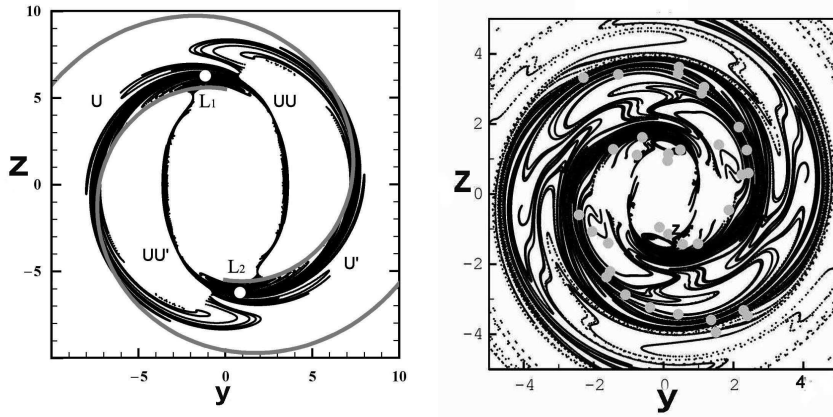


Fig. 5 (a) The projected unstable asymptotic manifolds from PL_1 and PL_2 in the configuration space (b) The “coalescence” of the invariant asymptotic manifolds from the PL_1 , PL_2 , $-1/1$, $-2/1$ and -41 families.

3 Resonant orbits and diffusion times

The planar orbits in a time independent rotating model have a fixed Jacobi constant (that we call “energy in the rotating frame”)

$$E_j = \frac{1}{2}u^2 + V(r, \theta) - \Omega_s J \quad (16)$$

where u is the velocity in the rotating frame, $V(r, \theta)$ is the potential in polar coordinates, J is the angular momentum and Ω_s is the angular velocity of the system.

If E_j is larger than the energy E_{jo} at the Lagrangian points L_1, L_2 , the orbits inside and outside corotation can communicate. If, however, $E_j < E_{jo}$ the orbits inside corotation cannot get outside and those outside corotation cannot get inside.

We consider a particular N-body system that was studied by Voglis and Stavropoulos (2005) and Voglis et al. (2006a) simulating a barred spiral galaxy and we separate the orbits in particular intervals $E_j \pm 10000$.

We consider first the N-body orbits that have energies in the interval $E_j = -1090000 \pm 10000$. In this energy level there are almost no regular orbits at all (see Fig. 3 of Har-soula et al. 2011). For these energies the orbits can move both inside and outside corotation. We integrate the orbits with initial conditions outside corotation for $100T_{hmct}$ (half-mass crossing times) and find the distribution of their q values (Fig. 6a).

The time interval $\Delta t = 100T_{hmct}$ is about one third of the Hubble time. During that time most resonant orbits are concentrated near the resonances $-2/1$ (outer Lindblad), $-1/1$ and $-2/3$. Only a few orbits have moved inside corotation ($q > 0$).

However after about one and a half Hubble time (Fig. 6b) the above resonances have fewer stars. Some stars have escaped, but a substantial proportion of stars have reached the inner resonances $3/1$ and $2/1$ (inner Lindblad). These stars are trapped near these resonances for very long times before escaping again outwards. On the other hand stars starting inside corotation remain close to the resonances $3/1$ and $2/1$ for more than 5 Hubble times before escaping outside the galaxy (without being trapped by the outer resonances). It must be pointed out here that the 2-body relaxation time

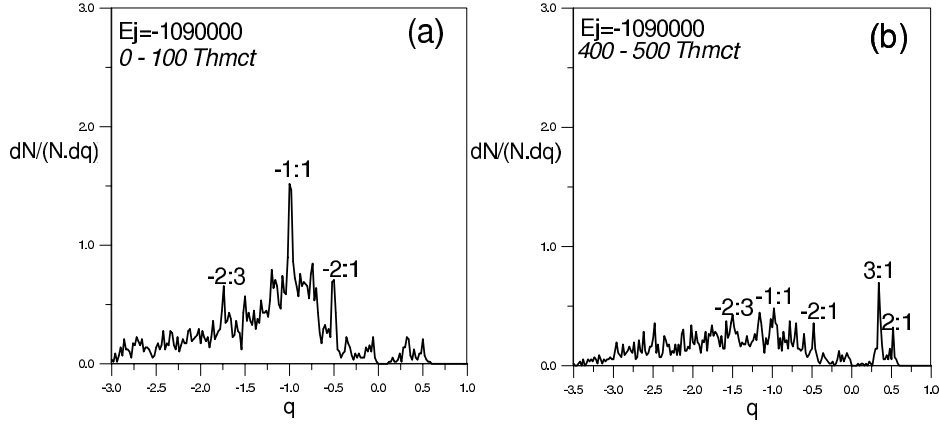


Fig. 6 The distribution of the frequency ratios q of N-body chaotic orbits having initial conditions outside corotation and belonging to the energy level $E_j = -1090000 \pm 10000$, when the orbits are integrated (a) for $100T_{hmct}$ (corresponding to $\approx 1/3$ Hubble time) and (b) from $400T_{hmct}$ to $500T_{hmct}$ (corresponding to $\approx 1,5$ Hubble time).

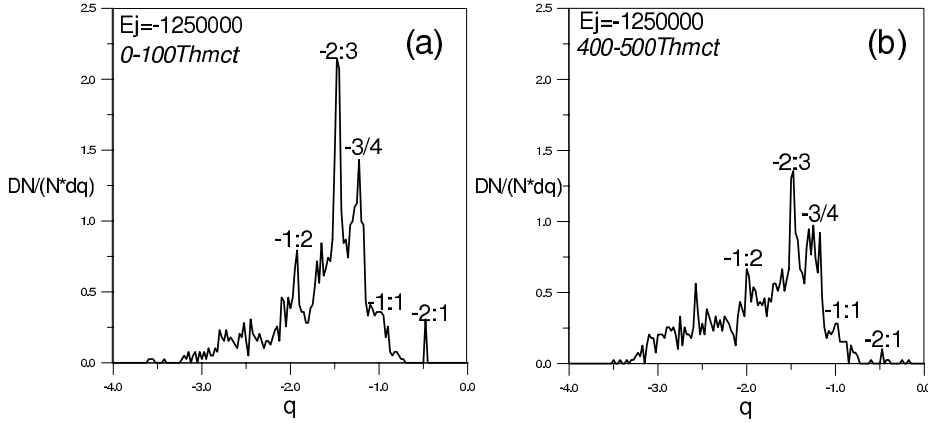


Fig. 7 The distribution of the frequency ratios q of N-body chaotic orbits having initial conditions outside corotation and belonging to the energy level $E_j = -1250000 \pm 10000$, when the orbits are integrated (a) for $100T_{hmct}$ (corresponding to $\approx 1/3$ Hubble time) and (b) from $400T_{hmct}$ to $500T_{hmct}$ (corresponding to $\approx 1,5$ Hubble time).

for galaxies is of the order of $10^6 - 10^7$ Hubble times, a time exceedingly longer than the diffusion time of the chaotic orbits in our N-body model.

Then we consider the stars outside corotation for an energy interval $E_j = -1250000 \pm 10000$, where again only chaotic orbits exist outside corotation. In this case the orbits with initial conditions outside corotation cannot enter inside corotation. In Figs. 7a,b the distribution of the q -values of these stars is given, for the time intervals $0 - 100T_{hmct}$ (Fig. 7a) and $400 - 500T_{hmct}$ (Fig. 7b). In this case there are no PL_1, PL_2 orbits, but we notice some important resonances like $-2/1$ (outer Lindblad), $-1/1, -3/4, -2/3$ and $-1/2$. After one and a half Hubble times (Fig. 7b) the number of resonant stars has decreased and many stars have escaped beyond the ends of the galaxy. Nevertheless there is still an appreciable number of stars close to these resonances. This is due

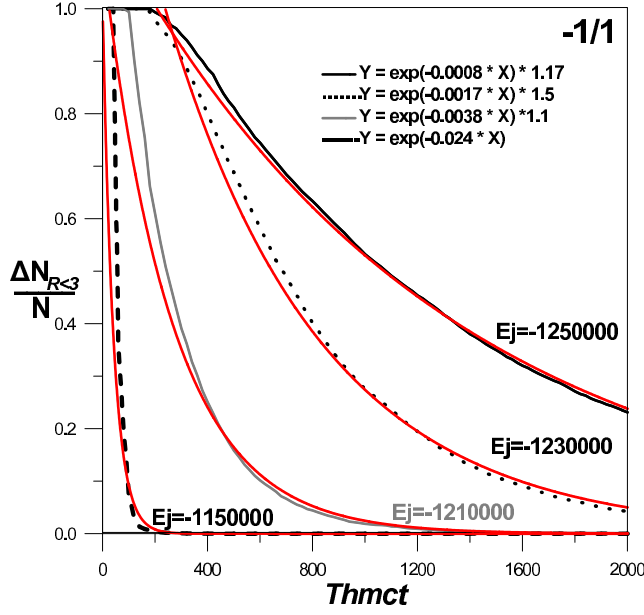


Fig. 8 The percentage $\Delta N_{R<3}/N$ of the orbits starting close to the $-1/1$ unstable periodic orbits that stay inside $R = 3r_{hm}$ as a function of time, in T_{hmct} , for four different energy levels, together with the corresponding exponential formulae (red curves).

to the stickiness phenomenon that lasts for very long times, before the stars escape from the galaxy. Comparing Figs. 6 and 7 we notice that in the case of chaotic orbits that are restricted in the area outside corotation, stickiness to resonances lasts for longer times in smaller values of Jacobi constants than in greater ones. Thus in general, the diffusion of chaotic orbits supporting the outer parts of the spiral arms outwards is more slow than the diffusion of orbits supporting more inner parts of the spiral structure. An example is given in Fig. 8 where the percentage of orbits starting close to the $-1/1$ unstable periodic orbits that stay located inside a radius $R = 3r_{hm}$ (which in fact confines the bound part of the galaxy, r_{hm} being the half mass radius of the system) is plotted as a function of time in T_{hmct} , for four different energy levels (Jacobi constants), namely for $E_j = -1250000$ (black solid curve), $E_j = -1230000$ (black dotted curve), $E_j = -1210000$ (gray solid curve) and $E_j = -1150000$ (black dashed curve). After 1 Hubble time ($\approx 300T_{hmct}$), 95% of the orbits with $E_j = -1250000$ is still located inside $R = 5r_{hm}$, while **90%** of the orbits with $E_j = -1230000$, 40% with $E_j = -1210000$ and only 0.1% of the orbits with $E_j = -1150000$ is still located inside $R = 3r_{hm}$. The functions $\Delta N_{R<3}/N$ versus T_{hmpt} for the various energy levels are approximated by exponentials of the form

$$\Delta N_{R<3}/N = \alpha \exp(-\lambda T_{hmpt}) \quad (17)$$

where α and λ take the values given in Table 1.

The values of α are close to $\alpha = 1$, while λ can be given by the approximate formula

$$\lambda = A \exp(\Lambda E) \quad (18)$$

Table 1 The values of α and λ in eq. (17)

E	α	λ
-1250000	1.17	0.0008
-1230000	1.50	0.0017
-1210000	1.10	0.0038
-1150000	1.00	0.0240

with $A = 1.8 \times 10^{15}$ and $A = 3.4 \times 10^{-5}$. Therefore the diffusion is much faster for larger Jacobi constants. On the other hand for smaller Jacobi constants the diffusion is slower, and most of the stars remain close to the outer parts of the spiral arms for more than a Hubble time.

In the case of chaotic orbits with initial conditions inside corotation or close to it, the diffusion happens quickly for an initial time interval corresponding to $\approx 1/3$ of the Hubble time (during which the spiral structure survives), while later on it is very slow (see Fig. 24 of Harsoula et al. 2011). We therefore conclude that the outer parts of the spiral structure of our N-body model survive for longer times than the inner parts of the spiral structure.

4 The role of asymptotic orbits

In previous papers (Harsoula and Kalapotharakos 2009, Harsoula et al. 2011) we have emphasized the role of stickiness of chaotic orbits along the unstable asymptotic manifolds of the unstable periodic orbits, in supporting the structure of the spiral arms. In what follows, we investigate the role of the 2-D asymptotic orbits, i.e. orbits having initial conditions on the unstable manifolds of the various unstable periodic orbits, in supporting the spiral structure of the model. Orbits in different energy levels support different parts of the spiral structure. This is obvious in Fig.9 where the isodensities of the N-body particles are plotted on the configuration plane of rotation, belonging to three different energy levels. More precisely, particles having values of Jacobi constant $E_j = -1090000 \pm 10000$ correspond to the envelope of the bar and the innermost parts of the spiral arms. Particles with values of Jacobi constant $E_j = -1150000 \pm 10000$ correspond to parts of the spiral arms that extend further beyond and finally particles with values of Jacobi constant $E_j = -1250000 \pm 10000$ correspond to the outermost part of the spiral arms. In Fig. 9d we plot the isodensities of particles belonging to all previous energy levels of Figs. 9a,b,c. The spiral structure of the galaxy is apparent.

In Harsoula et al. 2011 we studied the density distribution of 3-D orbits starting close to the various resonances. However we find similar results if we consider the projections of the various orbits on the plane of symmetry ($y-z$) of the galaxy (having the bar along the z-axis).

In Fig. 10 an example of the density distribution of 2-D asymptotic orbits of an unstable periodic orbit is plotted (in color) for a Jacobi constant $E_j = -1090000$, where the areas inside and outside corotation can communicate. More precisely in Fig.10a we plot (in color) 10000 asymptotic orbits of the 2/1 (or $x1$) family, having initial condition inside corotation, together with the unstable periodic orbit (in black). The corresponding time of integration of the orbits is \approx one and a half Hubble time. For an initial time interval equal to $\approx 1/5$ of the Hubble time, the chaotic orbits stay close to the periodic orbit, following its shape, but later on they are diffused outwards

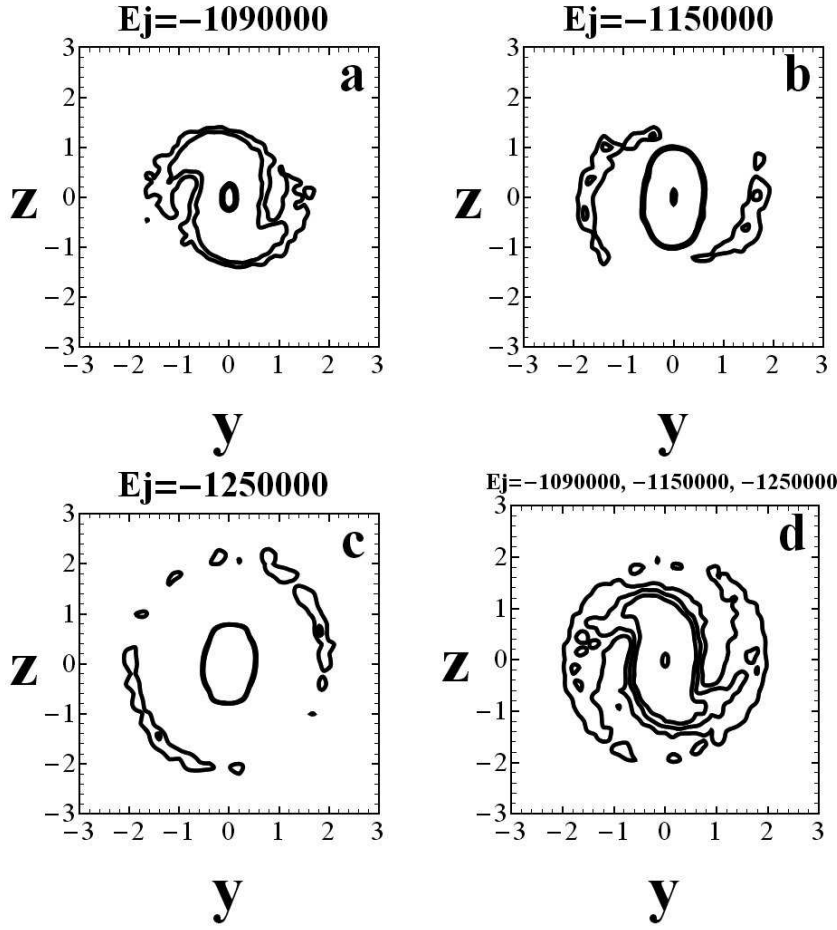


Fig. 9 The isodensities of the N-body particles belonging to three different energy levels, namely (a) $E_j = -1090000 \pm 10000$, where the areas inside and outside corotation can communicate (b) $E_j = -1150000 \pm 10000$, where the areas inside and outside corotation cannot communicate (c) $E_j = -1250000 \pm 10000$ where again the areas inside and outside corotation cannot communicate and (d) the isodensities of particles belonging to all previous three levels. The spiral structure is apparent here.

modulating the inner parts of the spiral structure. If we take the same number of orbits, with initial conditions not on the unstable manifold, but on a grid close and around the unstable periodic orbit having small deviations from it, on the (z, \dot{z}) surface of section (Fig. 10b), we find that the distribution of the orbits (in color) follows the inner parts of the spiral pattern, derived from the isodensities of the real N-body particles of the corresponding energy level (in black). This is an example of stickiness of chaotic orbits along the unstable asymptotic manifolds of the unstable periodic orbits. In fact the stickiness of chaotic orbits delays their diffusion outwards and is responsible for the survival of the spiral structure of the galaxy for more than 10 rotations of the bar (see Harsoula et al. 2011).

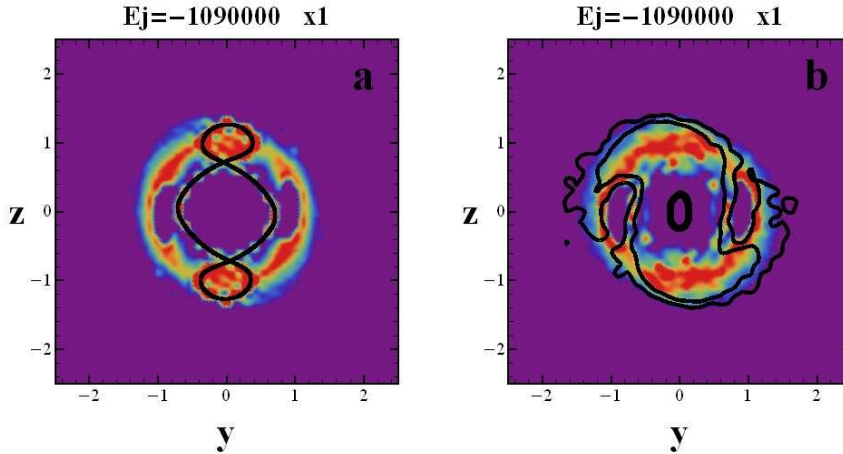


Fig. 10 (a) The density distribution of 10000 2-D asymptotic orbits (in color) belonging to the $2/1$ (or $x1$) family having initial conditions inside corotation and Jacobi constant $E_j = -1090000$. Superimposed is the orbit $x1$ plotted in black. (b) The density distribution of 2-D orbits having initial conditions on a grid close and around the unstable periodic orbit $2/1$ (in color). Superimposed, in black, are the isodensities of the real N-body particles belonging to the same energy level. The corresponding time of integration of the orbits is \approx one and a half Hubble time.

Similar results are found for the PL_1, PL_2 orbits near L_1, L_2 at the end of the bar, for the $-1/1$ orbits outside corotation and for the PL_4, PL_5 orbits around L_4 and L_5 .

Finally, in Fig. 11 we present the density distribution of the sticky chaotic orbits near the resonances $2/1, PL_1, PL_2, -1/1, -2/1$ superimposed with the isodensities of the real N-body particles belonging to the corresponding energy levels. We conclude that by using a sample of sticky chaotic orbits around a number of unstable periodic orbits inside and outside corotation in different energy levels, we are able to reproduce quite well the outer envelope of the bar and the spiral structure of the galaxy.

5 Conclusions

The main conclusions of our paper are the following:

- 1) Stickiness of chaotic orbits close to the unstable asymptotic manifolds of various periodic orbits delays the diffusion of these orbits outwards and therefore modulates the shape of the spiral structure of the galaxy for more than 10 rotations of the bar, corresponding to $1/3$ of the Hubble time.
- 2) Chaotic orbits that are limited outside corotation modulate the outer parts of the spiral structure for smaller values of Jacobi constant while orbits with greater values of Jacobi constant modulate the inner parts of the spiral structure. Moreover, in our N-body model, stickiness to resonances for smaller values of Jacobi constants lasts for longer times than stickiness for greater values of Jacobi constants.
- 3) Asymptotic orbits (having initial conditions on the unstable asymptotic curve of an unstable periodic orbit) stay located close to the periodic orbit for an initial interval of time, following the shape of this specific orbit, before diffusing from it and supporting the spiral structure. Chaotic orbits having initial conditions inside corotation modulate

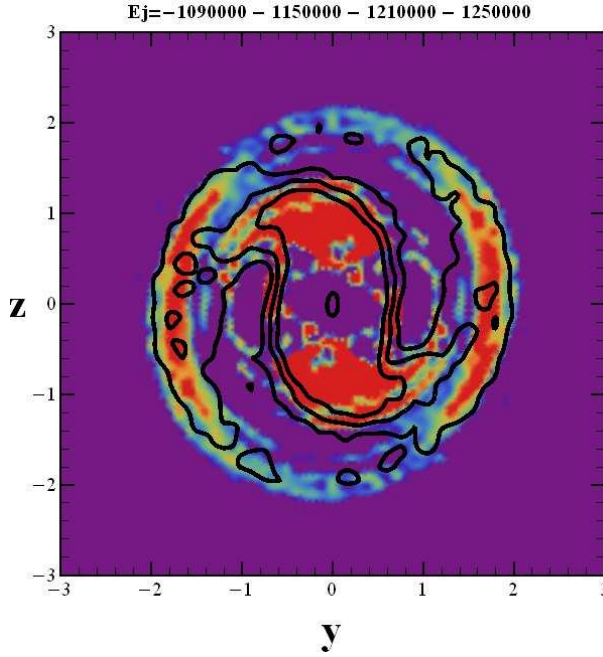


Fig. 11 The density distribution of the orbits starting close to the resonances $2/1, PL_1, PL_2, -1/1$ and $-2/1$ for various Jacobi constants, superimposed with the isodensities of the real N-body particles belonging to the corresponding energy levels (black curves).

the envelope of the bar and the innermost spiral structure during a time interval of fast diffusion ($\approx 1/3$ of the Hubble time) and then they are diffused outwards with much slower rates.

4) Using a sample of sticky chaotic orbits close to a number of unstable periodic orbits inside and outside corotation, in different energy levels, we are able to reproduce quite well the outer envelope of the bar and the spiral structure of the galaxy.

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